

Time-weighted Method and Capital-weighted Method for Calculating Returns

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ABSTRACT

In people's daily lives, they encounter numerous investment activities, nearly all of which aim at generating returns. A straightforward method to assess the effectiveness of returns is through the rate of return, which represents the interest rate that equalizes the present value of the invested funds with the present value of the recovered funds. This article takes a one-year investment as an example to introduce various methods for calculating the investment rate of return. The calculation method for investments exceeding one year is essentially the same as that for one-year investments, which can also be calculated annually and then the total rate of return can be comprehensively calculated. However, this article will not cover this for the time being.

Keywords: yield, capital-weighted method, time-weighted method.

1. INTRODUCTION

The calculation of some returns is relatively straightforward. For instance, if 100 yuan is invested at the beginning of the year, with no investments or withdrawals in between, and a total of 105 yuan is withdrawn at the end of the year, the return rate is 5%. However, if there are multiple capital investments and withdrawals within an investment period, the return cannot be calculated simply by subtracting the initial and final capital amounts. A more precise method is needed to calculate the return during this period.

1.1 Introduction to Capital-weighted Method

For example, Investor A invests 10,000 yuan in a project on January 1st. This project reports quarterly and allows for the addition or withdrawal of partial investments. On April 1st of the same year, the account balance becomes 10,200 yuan, and 5,000 yuan is deposited into the account. On July 1st of the same year, the account balance is 15,000 yuan, and 7,000 yuan is withdrawn. On October 1st of the same year, the account balance is 8,200 yuan, and 3,000 yuan is withdrawn. By the end of the year on December 31st, the account

balance is 5,500 yuan. For this investment process, we can use the capital weighted method or time weighted method to calculate the rate of return.

The core of the capital-weighted method is to focus on the macro changes in the amount of capital, without paying too much attention to the timing and quantity of capital investment. In this method, we introduce the following symbols:

A: Initial investment amount of the project;

B: Account balance at the end of the project;

I: Interest income during the investment process;
i: Rate of return on investment;

C_t: The amount invested at time *t* ($0 < t < 1$). If the investment is recovered at this time, the amount will be negative;

C: The total amount of new funds invested throughout the process, i.e., $C = \sum_t C_t$.

Therefore, there is the following capital balance formula

$$B = A + C + I \quad (1)$$

In addition, if the interest is calculated as the accumulated value of the principal and compounded, the following formula applies

$$I = iA + \sum_t C_t (1 + i)^{(1-t)} - C \quad (2)$$

In (2), the yield i can be solved through numerical calculation, but the calculation poses a certain degree of difficulty. For short-term investments with a term of less than one year, the difference between simple interest and compound interest is not significant, so the accumulation of interest rate can be assumed to be linear, that is

$$I = iA + \sum_t C_t i(1-t) \quad (3)$$

A simpler formula can be obtained from equation (3)

$$i = \frac{I}{A + \sum_t C_t (1-t)} \quad (4)$$

In the aforementioned investment, it is known that

$$\begin{aligned} A &= 10000 \text{ yuan,} \\ C_{1/4} &= 5000 \text{ yuan,} \\ C_{2/4} &= -7000 \text{ yuan,} \\ C_{3/4} &= -3000 \text{ yuan,} \\ C &= -5000 \text{ yuan,} \\ B &= 5500 \text{ yuan.} \end{aligned}$$

Substituting into equation (4) yields

$$\begin{aligned} i &= \frac{I}{A + \sum_t C_t (1-t)} \\ &= \frac{5500 - 10000 + 5000}{10000 + 5000 \times 0.75 - 7000 \times 0.5 - 3000 \times 0.25} \end{aligned}$$

$$B_k = (B_{k-1} + C_{k-1})(1 + j_k), \quad k = 1, 2, \dots, m. \quad (5) \quad j_k = \frac{B_k}{B_{k-1} + C_{k-1}} - 1, \quad k = 1, 2, \dots, m \quad (6)$$

The annual return rate i satisfies the following formula

$$1 + i = (1 + j_1)(1 + j_2) \cdots (1 + j_m)$$

namely

$$i = (1 + j_1)(1 + j_2) \cdots (1 + j_m) - 1 \quad (7)$$

In the aforementioned investment, it is known that

$$\begin{aligned} B_0 &= A = 10000 \text{ yuan,} \\ C_1 &= 5000 \text{ yuan,} \\ C_2 &= -7000 \text{ yuan,} \\ C_3 &= -3000 \text{ yuan,} \\ C &= -5000 \text{ yuan,} \\ B_1 &= 10200 \text{ yuan,} \\ B_2 &= 15000 \text{ yuan,} \\ B_3 &= 82000 \text{ yuan,} \\ C &= -5000 \text{ yuan,} \\ B &= 5500 \text{ yuan.} \end{aligned}$$

Substitute the data into the formula, we have

$$j_1 = \frac{10200}{10000} - 1 = 0.02$$

$$= 5.26\%$$

That is, using the capital weighted method, the annual return rate of the aforementioned investment can be approximately 5.26%.

1.2 Introduction to Time-weighted Method

Let's take a look at the time-weighted method. The core idea of the time-weighted method is to calculate the periodic return for each capital contribution or withdrawal, and then calculate the comprehensive return for the entire investment period. This method can also utilize the return to reflect the changes in capital, and the annual return is related to both the net amount of new capital invested and the specific investment timing.

Here, we only consider the scenario where the investment period is one year. Assuming there are $m-1$ capital movements within one year, the times are t_1, t_2, \dots, t_{m-1} , in sequential order, and the amounts invested are C_1, C_2, \dots, C_{m-1} , respectively. Let B_k be the account balance at time t_k and immediately before the investment of C_k , and j_k represent the actual return rate for the k th time period. Thus, we have

$$j_2 = \frac{15000}{10200 + 5000} - 1 = -0.0136$$

$$j_3 = \frac{8200}{15000 - 7000} - 1 = 0.025$$

$$j_4 = \frac{5500}{8200 - 3000} - 1 = 0.0577$$

$$\begin{aligned} i &= (1 + j_1)(1 + j_2) \cdots (1 + j_m) - 1 \\ &= (1 + 0.02)(1 - 0.0136)(1 + 0.025)(1 + 0.0577) - 1 \\ &= 9.08\% \end{aligned}$$

2. BRIEFLY ANALYZING THE RESULTS OF THE TWO METHODS

Using the time-weighted method, the researchers can obtain an annual return rate of approximately 9.08% for the aforementioned investment. The discrepancy between the two results lies in the fact that the 9.08% obtained through the time-weighted method represents the overall profitability of the project, independent of individual investor behavior, whereas the 5.26% calculated using the capital-weighted method

pertains specifically to the profitability of investor A in this project. According to the time-weighted method, the project yielded profits in the first quarter, benefiting investors; however, in the second quarter, despite the project being in a loss position, investors injected new funds at the beginning of this stage, resulting in a peak principal amount during this period, which consequently led to significant losses for investors. In the third and fourth quarters, despite the project generating good profits, investors withdrew part of their investments, thus missing out on the investment returns during this period. Due to these suboptimal investment decisions, individual investors' returns fell short of what the project could have yielded.

Using the time-weighted method, the annual return rate of the aforementioned investment can be approximately calculated as 9.08%. The difference between the two results lies in the fact that the time-weighted method's 9.08% includes the time-

$$i = \frac{I}{A + \sum_t C_t(1-t)} = \frac{16270.08 - (10000 - 5000 + 5000 + 5000)}{10000 - 5000 \times 0.75 + 5000 \times 0.5 + 5000 \times 0.25}$$

$$= 12.70\%$$

The investment performance of Investor B is significantly better than that of the project itself, which is due to the timely investment or withdrawal of funds during the process. However, this requires a highly accurate judgment of the market by individuals, which is not easily achievable for ordinary investors. Whether it's Investor A or Investor B, there are relatively significant differences in investment results and projects, which is due to the large fluctuations in the market across different quarters. When the investment market itself is relatively stable, or when the proportion of mid-course capital changes is relatively small, the individual investment results and the overall project will not differ greatly.

3. CONCLUSION

The time-weighted method, with compound interest at its core, calculates the overall project returns, suitable for evaluating manager capabilities and ensuring horizontal comparability of products. The capital-weighted method, framed around the internal rate of return, reflects the efficiency of funds throughout the entire cycle, closely aligning with the actual holding returns of individual investors. These two methods are not substitutes for each other, but complementary tools based on different evaluation objectives. The differences in

weighted phased returns. For the same investment project, one only needs to understand the investment process of other investors, rather than the balance at each stage, to derive the individual return rate of the investor.

For example, investor B invested 10,000 yuan in the project on January 1st, withdrew 5,000 yuan of investment on April 1st, and invested another 5,000 yuan on July 1st and October 1st respectively. Calculate the investor's personal return rate.

Based on the overall project's revenue, the investor's year-end balance can be calculated. The specific process is as follows:

$$10000 \times 1.02 = 10200$$

$$(10200 - 5000) \times (1 - 0.0136) = 5129.28$$

$$(5129.28 - 5000) \times (1 + 0.025) = 10382.512$$

$$(10382.512 + 5000) \times (1 + 0.0577) = 16270.08$$

Using the time-weighted method to calculate personal return rate

results stem from different evaluation subjects and objectives: time-weighted is suitable for evaluating manager performance, while capital-weighted is more appropriate for assessing investor experience and account returns.

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